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Comments on "Iterative and Power Series Solutions for the Large Deflection of an Annular Membrane"

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Introduction

In a recent technical note,[1] Pifko and Goldberg applied the Föppl-Hencky large deflection membrane equations to a uniformly loaded annular membrane, fixed at the outer boundary and supported at the inner boundary, and reduced the governing equations to the following nonlinear differential equation and boundary conditions:

$$f^2(d^2f/d\xi^2) = -\tfrac{1}{32}(\lambda^2 - \xi)^2 \tag{1}$$

$$2df/d\xi - (1 + \nu)f = 0 \text{ at } \xi = 1 \tag{2}$$

$$f = 0 \text{ at } \xi = \lambda^2 \tag{3}$$

where f is a nondimensional stress function defined in Ref. 1, $\xi^{1/2}$ the nondimensional radial distance, λ the nondimensional inner boundary radius, and ν is Poisson's ratio.

These authors constructed a power series solution of Eq. (1) about the point $\xi = \lambda^2$, satisfying Eq. (3), of the form

$$f(\xi) = (a_1{}^4\Phi/4)[1 - \Phi - \tfrac{2}{3}\Phi^2 - \tfrac{13}{18}\Phi^3 - \tfrac{17}{18}\Phi^4 - \tfrac{37}{27}\Phi^4 - \tfrac{1205}{567}\Phi^6 \ldots] \tag{4}$$

where

$$\Phi = (\xi - \lambda^2)/a_1{}^3 \tag{5}$$

They also provided a table of values of a_1 for various values of λ for $\nu = 0.3$ which make Eq. (4) satisfy Eq. (2).

The purpose of this note is first to point out that Eqs. (1–5) are a limiting form of a set of equations obtained by the writer in Ref. 2 and that the series in brackets in Eq. (4), including one additional term, can be readily obtained from Eqs. (29) and (31) of Ref. 2, and second to show how, by a simple calculation, a table of a_1 vs λ for any value of ν may be constructed from Table 1 of Ref. 1.

Derivation of Results

We first note that the series in brackets must be a universal function since its coefficients are independent of the parameters λ and ν, which enter into the boundary conditions given by Eqs. (2) and (3). Indeed, if we take Φ as a new independent variable,

$$g = (4/a_1{}^4)f \tag{6}$$

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as a new dependent variable, and set

$$a_1 = 4df/d\xi \text{ at } \xi = \lambda^2 \tag{7}$$

then Eqs. (1–3) go over into the following parameter-free initial value problem:

$$g^2(d^2g/d\Phi^2) + 2\Phi^2 = 0 \tag{8}$$

$$g = 0,\ dg/d\Phi = 1 \text{ at } \Phi = 0 \tag{9}$$

Equation (2) now reads

$$dg/d\Phi - 2a_1{}^3(1 + \nu)g = 0 \text{ at } \Phi = (1 - \lambda^2)/a_1{}^3 \tag{10}$$

and the method by which this equation is to be satisfied will be discussed presently.

In Ref. 2, the equations governing the large deflections of a normally loaded, spinning, elastic membrane were reduced to the following differential equation and initial conditions:

$$y^2(d^2y/dx^2) + kx^2 + 2y^2 = 0 \tag{11}$$

$$y = 0,\ dy/dx = 1 \text{ at } x = 0 \tag{12}$$

If the following change of variable is introduced,

$$x = 2\Phi/k \qquad y = 2g/k \tag{13}$$

then Eqs. (11) and (12) read

$$g^2(d^2g/d\Phi^2) + 2\Phi^2 + (4/k)g^2 = 0 \tag{14}$$

$$g = 0,\ dg/d\Phi = 1 \text{ at } \Phi = 0 \tag{15}$$

In the limit as $k \to \infty$, Eqs. (14) and (15) become identical to Eqs. (8) and (9).

A power series solution of Eq. (11), subject to Eqs. (12), is given by Eqs. (27, 29, and 30) of Ref. 2, where the first seven coefficients of this series are explicitly calculated. If the transformations given by Eqs. (13) are introduced into this power series solution and the limit taken as $k \to \infty$, we obtain terms identical to those in brackets in Eq. (4) plus the one additional explicit term

$$-(219241/63504)\Phi^7 \tag{16}$$

Returning to Eq. (10), we must, in order to satisfy this equation, pick an arbitrary value of Φ, say Φ_0, calculate the corresponding value of $(dg/d\Phi)_0$ and g_0, solve for the value of $a_1{}^3$ for which Eq. (10) is satisfied, and then see what λ turns out to be from the condition

$$\Phi_0 = (1 - \lambda^2)/a_1{}^3 \tag{17}$$

The foregoing analysis indicates how a table of a_1 vs λ for *any* value of Poisson's ratio ν may be constructed once a table for a *particular* value of ν has been constructed. Specifically, if we let λ and a_1 correspond to $\nu = 0.3$ and Λ and A_1 correspond to arbitrary values of ν, then for any value of Φ_0, the following two relations must hold:

$$1.3a_1{}^3 = (1 + \nu)A_1{}^3, (1 - \lambda^2)/a_1{}^3 = (1 - \Lambda^2)/A_1{}^3 \tag{18}$$

or

$$A_1 = \left(\frac{1.3}{1 + \nu}\right)^{1/3} a_1, \qquad \Lambda = \left[1 - \frac{1.3}{1 + \nu}(1 - \lambda^2)\right]^{1/2} \tag{19}$$

From Eqs. (19), a table of A_1 vs Λ for any ν can be constructed with a_1 vs λ given by Table 1 of Ref. 1.

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